Query complexity and local search problems

Vsevolod Oparin

St. Petersburg University of
Russian Academy of Science

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Query complexity

Type of problem

Let D be a domain and $f : D \to O$ be a function.

- The Respondeer knows a word $x \in D$.
- The goal of the Questioneer is to find $f(x)$.

Questioneer can ask some type of questions.

The function f can be replaced by a relation $R \subseteq D \times O$. So the goal of the Questioneer is to find $y \in O$ s.t. $R(x, y)$.

Example

Guess the Number!

Guess the Number

Problem

Let D be a set of n-bit strigns s.t. $D = \{1^k 0^{n-k} \mid 0 \le k \le n\}$ and $f(x) =$ number of 1 in x. The Questioneer is allowed to make request for the value of any bit of x. The goal is to find $f(x)$ in the minimal number of questions.

Theorem

The number of questions is $\Theta(\log n)$.

Arrows

The alhabet is $\Sigma = \{\nearrow, \uparrow, \nwarrow, \leftarrow, \swarrow, \downarrow, \searrow, \rightarrow\}$

Constraints

- Every arrow on the border points inside.
- Every two neughbouring arrows differ in at most 45^o.

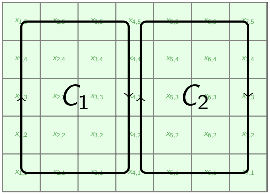
↘	↘	↘	↘	↓	↙	↙
↘	↓	↓	↓	↓	↓	↙
→	↘	↘	↓	↙	↙	←
→	→	→	↙	←	←	←
→	↗	↗	↑	↖	↖	←
↗	↗	↗	↑	↖	↖	←
↗	↑	↑	↑	↑	↑	↖

Theorem

There is always at least one contradiction.

Theorem

There is the strategy which allows to find a contradiction in $O(n)$ queries.



Lower bound

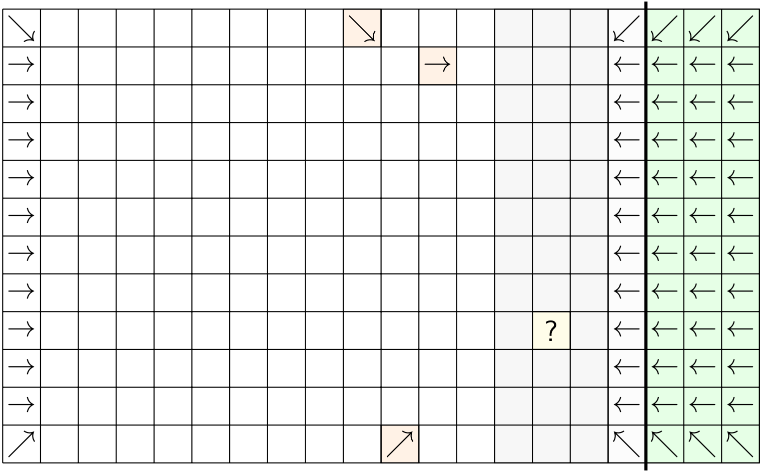
Theorem

In worst-case, every strategy makes $\Omega(n)$ queries.

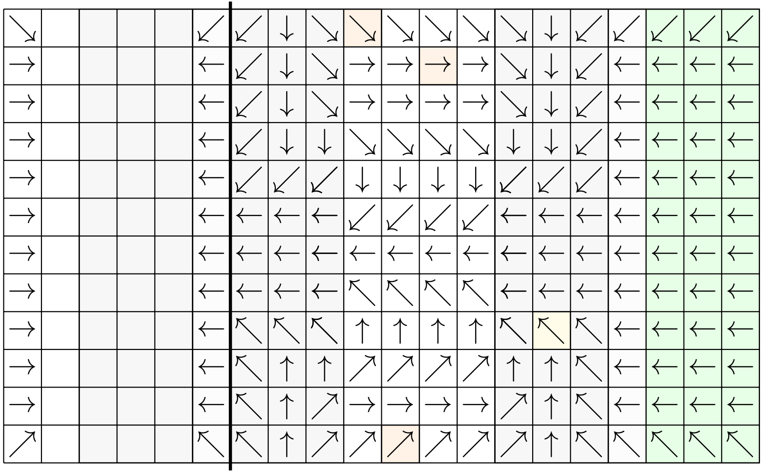
Adversary technique

We provide the strategy for the respondeer which allows to build a hard example for any deterministic algorithm.

Adversary strategy



Adversary strategy



Thank you!

Any questions?